NUMERICAL MODEL FOR CREEP OF PLANE CONCRETE IN COMPRESSION AND TENSION

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Abstract
This paper presents a numerical model for the analysis of concrete creep in plane deformation. Creep deformations of concrete were determined by means of correcting the modulus of elasticity of concrete in the principal directions for the current state of stresses. The correction of the modulus of elasticity, in an direction, depends on the current creep coefficient and the current elasticity modulus of concrete in this direction. With this numerical model it is possible to analyze the creep deformations of reinforced and prestressed concrete structures. Based on comparison of the results obtained by this numerical model and the numerical and experimental results of other authors, a verification of the numerical model was done.

Keywords: numerical model, modulus of elasticity, creep, shrinkage, reinforced concrete.

1. INTRODUCTION
Realistic assessment of the behavior of reinforced and prestressed concrete structures in real conditions necessarily requires the involvement of time-dependent deformations of concrete, reinforcement, and prestressing tendons. This is primarily related to the creep and shrinkage deformation of concrete and relaxation of steel. The influence of time-dependent deformation of concrete is reflected in the increase of deformations, stress redistribution and the state of cracks in concrete.

Creep of concrete belongs to the viscous deformations whose mechanism and development is not fully understood. Creep occurs in the area of compressive and tensile stresses in the concrete. Since the tensile strength of concrete is significantly lower than the compressive strength the creep in tension is not so significant. Analysis of reinforced and prestressed concrete structures, especially with large spans and heights, is not complete without the inclusion of time-dependent deformations of concrete.

There is a large number of concrete creep models and most of them are based on empirical expressions that define certain parameters that affect the creep of concrete [1]. Experimental models are mostly simple and mapping their results on complex real structures is not always appropriate. Therefore, numerical models
can provide important information about the behavior of reinforced and prestressed concrete structures in real conditions.

The adopted model of concrete includes all the essential nonlinear effects of concrete, such as yielding, crushing and cracking. Shrinkage model is taken from EC2, while for analysis of creep deformation a new numerical method is developed, which is based on the correction of the concrete stiffness matrix at the level of each integration point over time.

II. CONCRETE CREEP MODEL

The analysis of concrete creep deformations is not easy, not only because of the complexity of concrete properties, but also because the concrete is regularly used in combination with a reinforcement or prestressing steel. The development and magnitude of creep deformations are affected by a large number of factors, such as the composition of concrete mix (aggregate, cement and water, the water-cement ratio, the consistency of concrete mix), humidity and environment temperature, the beginning of first loading of structure, cross-sectional shape, conditions and the type of stresses and so on.

Most models of concrete creep refers to the determination of longitudinal deformations of concrete under uniaxial compressive stress. These models are then usually mapped to multiaxial states by the analogy with the classical theory of elasticity. Furthermore, most authors take Poisson's ratio of creep as constant and equal to the elastic Poisson's ratio. However, few experimental studies of concrete creep under multiaxial compressive stresses, show that the corresponding Poisson's creep ratio is not constant over time. Its initial value is less than in material elastic range. It depends on character of multiaxial stress state and therefore is not isotropic. Adopting a constant Poisson's ratio can lead to an underestimation of creep deformation up to 40% [2].

This numerical model is based on the correction of the current elasticity modulus of concrete in the direction of principal stresses, as a function of concrete creep coefficient. The creep coefficient is taken according to EC. Determination of current elasticity modulus of concrete, for the corresponding values of principal stresses at some time instant, is done via the stress and strain relationship for concrete in compression. It is a secant modulus of elasticity in two main directions.

For a given load level the principal stresses are known in all integration points, at the initial time point $t_0$. With the help of these stresses and adopted stress-strain relationship of concrete in compression is possible to determine the current secant elasticity modules for each principal direction. After that, the correction of the current elastic modulus by the creep coefficient of concrete is done. Thus the local orthotrophy of concrete is introduced at the level of the associated area of each integration point, which is reflected on the material matrix, or finally, the stiffness matrix of a finite element of concrete. Therefore, the material matrix has to be mapped from the material in the main axes, resulting in an orthotropic concrete model. This process is shown in Fig. 1.
For the case of plane stress state stress-strain relationship is:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{E} & -\nu & 0 \\
-\nu & \frac{1}{E} & 0 \\
0 & 0 & \frac{1}{G}
\end{bmatrix} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}, \tag{1}
\]

where \( E \) is initial concrete modulus, and \( \nu \) is Poisson's ratio.

By generalization of equation (1) for orthotropic materials, the following equation can be written:

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{E_1} & -\nu_{12} & 0 \\
-\nu_{21} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}, \tag{2}
\]

with
- \( E_1, E_2 \) elasticity modulus in principal material directions,
- \( \nu_{12}, \nu_{21} \) Poisson’s coefficients i plane 1-2,
- \( G_{12} \) shear modulus in plane 1-2.
For the shear modulus of orthotropic materials a few expressions have been proposed [3]:

\[ G_{12} = \frac{\sqrt{E_1E_2}}{2(1+\nu_1\nu_2)}, \]  
(5)

\[ G_{12} = \frac{E_1 + E_2}{4\left[1 + \frac{1}{2}(\nu_1 + \nu_2)\right]}, \]  
(6)

\[ G_{12} = \frac{E_1 + E_2 - 2\sqrt{\nu_1\nu_2E_1E_2}}{4(1-\nu_1\nu_2)}. \]  
(7)

Expression (5) is empirical, while the expression (7) is proposed in [4], and it is analogous to expression for the shear modulus of cracked concrete. The expression (5) is obtained taking the mean values of \( E_1 \) and \( E_2 \), and also of \( \nu_1 \) and \( \nu_2 \). Expression (8) is acceptable in the cases where none of the principal strain is not dominant, which means that it is applicable to concrete without cracking.

By inversion of equation (2) the conventional manner is obtained for expressing stress over strain:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
\frac{E_1}{c} & \frac{v_{12}E_2}{c} & 0 \\
\frac{v_{21}E_1}{c} & \frac{E_2}{c} & 0 \\
0 & 0 & G_{12}
\end{bmatrix}
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\gamma_{12}
\end{bmatrix},
\]  
(8)

During the incremental-iterative procedure in nonlinear structural analysis stresses and deformations are known in all integration points of the structure. The principal stresses are obtained from global stresses according to the expression (11). Also, for this level of stresses it is possible to determine the state of concrete strains, and therefore apply the appropriate procedure depending on whether the concrete is in the elastic range, the flow field, the tension softening, or there is an appearance of cracks in certain areas.

In the formulation of the model for the analysis of the creep deformation of concrete it is necessary to determine the current elastic modulus \( E(t) \). To make this possible it is necessary to adopt a 1D diagram of concrete in compression. A Hogenstad's parabola of concrete in compression is adopted, and shown in Fig. 2.
where \( c = 1 - \nu_{12}\nu_{21} \), and \( \frac{\nu_{12}E_2}{c} = \frac{\nu_{21}E_1}{c} \).

Finally, for the plain stress, orthotropic material matrix is:

\[
D_{m} = \begin{bmatrix}
\frac{E_1}{c} & \frac{\nu_{12}E_2}{c} & 0 \\
\frac{\nu_{21}E_1}{c} & \frac{E_2}{c} & 0 \\
0 & 0 & G_{12}
\end{bmatrix} = S^{-1}, \quad (9)
\]

Similar material matrix is defined for the state of plane strain and axially symmetric state of stresses.

**Correction of concrete elastic modulus**

In general, the loads can be applied at any angle relative to the main axis. Therefore, it is necessary to transform the material matrix from the material to the global coordinate system. The transformation is performed by the following transformation matrix:

\[
T = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2\sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & -2\sin \theta \cos \theta \\
-\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}. \quad (10)
\]

Principal stresses and directions are given by familiar expressions from theory of elasticity:

\[
\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}},
\]

\[
\theta = \frac{1}{2} \arctan \frac{2\tau_{xy}}{\sigma_x - \sigma_y}. \quad (11)
\]

Based on the known principal stresses the concrete deformations can be determined from compression diagram. From the relationship between the current principal stress in a direction and the corresponding current deformation, obtained from the diagram of concrete in compression, current secant modulus of concrete, in that direction can be obtained.

**III. PREPARE YOUR PAPER BEFORE STYLING BEFORE YOU BEGIN TO FORMAT YOUR PAPER, FIRST WRITE AND SAVE**
Figure 2. Hogenstad’s parabola

Principal stresses $\sigma_{i,2}(t_0), \sigma_{i,2}(t_1), \ldots, \sigma_{i,2}(t_n)$ in each time instant are obtained from numerical analysis.

After determining the current elasticity modulus of concrete is done their correction is performed according to defined model for the effective elasticity modulus in EC 2:

$$ E^k(t) = \frac{E'(t)}{1 + \phi(t, t_0)} = E'(t) \cdot B(t), \quad (12) $$

where $E^k(t)$ is corrected elasticity modulus of concrete, and $E'(t)$ is current elasticity modulus. Time function $B(t) = \frac{1}{1 + \phi(t, t_0)}$ depends on creep coefficient $\phi(t, t_0)$, and it is the same for each principal direction in some instant of time. This assumption is reasonable, although there is a choice of different time functions for different states of stresses.

These corrected modulus are included in the orthotropic material matrix, which has the following form for time $t_i$:

$$ D_{ort}(t_i) = \begin{bmatrix} E^k_{11}(t_i) & \frac{1}{2} & \frac{1}{2} \\ E^k_{22}(t_i) & E^k_{12}(t_i) & 0 \\ \frac{1}{2} & \frac{1}{2} & G_{12} \end{bmatrix}, \quad (13) $$

The previous orthotropic material matrix has to be transformed from the material to the global axis using transformation matrix given by (10). Thus, the an orthotropic material matrix, in global coordinates, has the form:

$$ D = TD_{ort}T^T. \quad (14) $$

This matrix is now entering into the stiffness matrix of finite element, and is valid for an assigned area of a given integration point. In the case when principal stress at the integration point is tensile, reduction of elastic modulus of concrete is performed in same way as in the case of compressive stress if there are no cracks. If there are cracks reduction of elastic modulus is performed according to the adopted model of tension softening.

**NUMERICAL EXAMPLES**

**Example 1.**

In this example, a comparison between the results of the numerical model of creep with experimental results published in the papers [5, 6] is given. Four reinforced concrete beams have experimentally tested, with constant load for a period of six months. The geometry and the load of beam are shown in Fig. 3, while the informations necessary for the analysis are shown in Table I. Two of the four beams are reinforced with symmetrical reinforcement (B1 and B2), while the remaining two beams (A1 and A2) have reinforcement only in the lower zone of beam. In this example, beams A1 and B2 are analyzed, both having the same geometry, but with a different reinforcement.
Figure 3. Geometry and loading of beam

<table>
<thead>
<tr>
<th>TABLE I. GEOMETRY AND MATERIALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>$h [mm]$</td>
</tr>
<tr>
<td>$d' [mm]$</td>
</tr>
<tr>
<td>$d [mm]$</td>
</tr>
<tr>
<td>$b [mm]$</td>
</tr>
<tr>
<td>$l [mm]$</td>
</tr>
<tr>
<td>$A_x [mm^2]$</td>
</tr>
<tr>
<td>$A_y [mm^2]$</td>
</tr>
<tr>
<td>$P [kN]$</td>
</tr>
<tr>
<td>$M [kNm]$</td>
</tr>
<tr>
<td>$f_{cyl.} [MPa]$</td>
</tr>
<tr>
<td>$E_x [GPa]$</td>
</tr>
<tr>
<td>$f_y [MPa]$</td>
</tr>
<tr>
<td>$E_y [GPa]$</td>
</tr>
</tbody>
</table>

Beams are discretized with 45 elements and 172 nodes, whereby the height of beams is divided into three elements.

Table II gives a comparison of displacements at the middle beam span after 180 days. It indicates well agreement with the experimental results given in [6], although it is quite rough finite element mesh.

<table>
<thead>
<tr>
<th>TABLE II: COMPARISON OF RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>$\Delta [mm], [9]$</td>
</tr>
<tr>
<td>$\Delta [mm], [10]$</td>
</tr>
<tr>
<td>$\Delta [mm], BS 8110$</td>
</tr>
</tbody>
</table>
Example 2.

Gilbert and Nejadi [7] experimentally studied the long-term behavior of reinforced concrete beams and slabs. The goal of the experimental analysis was primarily to study the impact of time-dependent deformation of concrete on the development and propagation of cracks. A total of six beams and six slabs were studied, while their behavior was monitored over a period of 400 days. Beams and slabs are loaded after 14 days. The geometry and load of beams “a” are selected so that the maximum moment is approximately 50% of the limit moment value, while for the beams “b” maximum moment was approximately 30% of its limiting value. Environmental conditions in which the samples are stored are not mentioned in [7, 8], but the results of tests of concrete creep samples are given. Also, the shrinkage deformations are measured on the unloaded samples of similar dimensions as the tested beams and plates.

Over time, the propagation of cracks due to bending and displacement of the mid-span of beams were monitored. Beams are discretized with 180 2D elements of concrete, while the reinforcement was discretized with 36 three-noded 1D finite elements. In this paper, beams B2-a, B2-b, B3-a and B3-b are analyzed. The geometry and loads of beams are shown in Fig. 4., while Table III presents the mechanical properties of concrete. A bilinear elasto-plastic relationship for reinforcement is adopted for all beams, with yield strength $f_y = 500.0MPa$, and modulus of elasticity $E_s = 200.0GPa$.

![Figure 4. Geometry and loading of beam](image)

**TABLE III: CONCRETE PROPERTIES**

<table>
<thead>
<tr>
<th>Mechanical properties [MPa]</th>
<th>Age [days]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>18.3</td>
</tr>
<tr>
<td>Flexural tensile strength</td>
<td>3.7</td>
</tr>
<tr>
<td>Indirect tensile strength</td>
<td>2.0</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>22820.0</td>
</tr>
</tbody>
</table>

Beams B2-a and B2-b are the same except for the load. Beam B2-a was loaded with forces $P = 18.6kN$, while the beam B2-b with forces $P = 11.8kN$. Geometry, reinforcement and load of beams are given in Table IV.
TABLE IV: GEOMETRY, REINFORCEMENT AND LOADS

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Beam</th>
<th>$A_s$ [mm$^2$]</th>
<th>$n_s$</th>
<th>$\Phi_s$ [mm]</th>
<th>$b$ [mm]</th>
<th>$d$ [mm]</th>
<th>$P$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[7]</td>
<td>B2-a</td>
<td>400.0</td>
<td>2</td>
<td>16</td>
<td>250</td>
<td>333</td>
<td>37.0</td>
</tr>
<tr>
<td></td>
<td>B2-b</td>
<td>400.0</td>
<td>2</td>
<td>16</td>
<td>250</td>
<td>333</td>
<td>23.5</td>
</tr>
<tr>
<td></td>
<td>B3-a</td>
<td>600.0</td>
<td>3</td>
<td>16</td>
<td>250</td>
<td>333</td>
<td>54.0</td>
</tr>
<tr>
<td></td>
<td>B3-b</td>
<td>600.0</td>
<td>3</td>
<td>16</td>
<td>250</td>
<td>333</td>
<td>30.0</td>
</tr>
</tbody>
</table>

In [7], cylindrical samples are tested, at stress of 5MPa, for obtaining creep coefficients over time. The values of these coefficients are given in Table V, and their comparison with the creep coefficients obtained according to EC2, which were used in this work.

The Fig. 5 shows a comparison of the mid-beams displacements of beams B3-a and B3-b, with results of experiments and numerical results given in [12]. From the results it is evident that this numerical model gives a real result that correlate well with the experimental results. The Fig. 6 shows a comparison of the displacements of beam B2-a with results of experiments and numerical results given in [11]. It is evident that this numerical model gives good results, which is also confirmed by the data given in Table 6, where a final displacements after 400 days are given.

TABLE V: CREEP COEFFICIENTS

<table>
<thead>
<tr>
<th>Experiment [3,4]</th>
<th>Numerical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time [days]</td>
<td>14 16 21 53 39</td>
</tr>
<tr>
<td>creep coefficient</td>
<td>0.0 0.1 0.3 0.9 1.7</td>
</tr>
</tbody>
</table>

Figure 5. Mid-span deflections of beams B3-a, and B3-b
Displacements [mm]  

Time [days]  

B2-a, Num. model with shrinkage  
B2-a, Num. model without shrinkage  
B2-a, Exp. [7]  

Figure 6. Mid-span deflections of B2-a beam  

TABLE VI: MID-SPAN DEFLECTIONS AFTER 400 DAYS  

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Mid-span deflection [mm]</th>
<th>measured</th>
<th>numerical</th>
<th>[13]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>14</td>
<td>400</td>
<td>14</td>
</tr>
<tr>
<td>[7]</td>
<td>B2-a</td>
<td>-5.0</td>
<td>-12.4</td>
<td>-4.95</td>
</tr>
<tr>
<td></td>
<td>B2-b</td>
<td>-2.1</td>
<td>-7.9</td>
<td>-2.04</td>
</tr>
<tr>
<td></td>
<td>B3-a</td>
<td>-5.8</td>
<td>-13.3</td>
<td>-5.81</td>
</tr>
<tr>
<td></td>
<td>B3-b</td>
<td>-2.0</td>
<td>-7.9</td>
<td>-2.2</td>
</tr>
</tbody>
</table>

CONCLUSIONS  

In this paper, the problem of time-dependent deformations of concrete was analyzed, namely the creep and shrinkage. A new numerical model for the analysis of creep deformation of concrete in plane is presented, based on the correction of its elastic modulus in the principal directions, for the current state of principal stress for each time instant.  

From the results of some examples it can be concluded that the model provides physically acceptable solutions, and possesses all the features of an efficient numerical procedure. Comparing the results of this model with the reference solutions in the literature, and the measurements performed in the field, it can be concluded that the model is primarily functional, but also rational in terms of its mathematical formulation.  

REFERENCES  