LIFT FORCE ON A CYLINDER IN A SHEAR CURRENT

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Abstract
An apparently new formula is derived for the lift force on a cylinder in a shear current: numerical constant times the mean flow speed far from the cylinder times the constant fluid density times the current shear. Measurements are needed for comparison with the formula. Inspiration for obtaining the formula came from a sailor's experience in a submarine rising through the wave zone, as related to the author.

Key Words: Lift force equation, cylinder in a shear current

1. Introduction

When I began teaching the waves course at the Graduate School of Oceanography, University of Rhode Island, an extra duty given to me was to teach a basic oceanography class at the submarine base in Groton Connecticut. During the coffee break of a particular class, the subject of which was most likely ocean waves, a student came up to me and told a bit from his experience. A submarine can comfortably ride out storms at sea by going down to or below the depth of wave influence, which is comparable to a characteristic wavelength (say at least 500 ft). However, if some emergency should arise such that the submarine must surface to fix the problem, it could be dangerous because control of the ascent is lost within the wave zone. All features of ocean waves diminish with increasing depth down from the sea surface, including the particle velocity of the waves. Therefore, from the top of the horizontally traveling submarine to the bottom there is a difference in particle speed: faster on top and slower on the bottom. Then by Bernoulli's law a noticeable net upward suction on the submarine is produced (where the speed is greatest the pressure is least).

To my knowledge nobody has published an algebraic calculation of the lift force on a cylinder embedded in a vertically sheared current. Therefore, an estimation of such a force is given below. One seemingly logical place to look for a previously worked out solution of the present problem is within the body of dynamic fluid motion founded on the assumption of irrotationality. Uniform flow past a cylinder (or a sphere) will be found treated there in the text books, of course, but not a cylinder immersed in a steady shear flow. Such an apparent omission is reasonable when it is remembered that the mean shear flow contains vorticity, in violation of the irrotational assumption.

2. Calculation

Given are the vertical shear in a horizontal current dU/dz of constant density $\rho$and the radius of the cylinder R whose long axis is horizontal and perpendicular to the flow, and there is no spin of the cylinder about this axis. Gravity is neglected so the “vertical” direction has no special meaning. Assuming that U increases with increasing z; the flow relative to the cylinder is greater at the cylinder’s top and smaller at its bottom. At the top let the flow speed be $V_0 + \delta U$ and at the bottom $V_0 - \delta U$, where V_0 is a given speed provided to the problem, either from theory or from measurements. For example, if the flow were uniform (no shear) with speed U far from the cylinder, at the top and bottom of the cylinder the relative speed is 2U according to irrotational theory. Also δU is short for (dU/dz)R. Idealizing by neglecting turbulence makes the following concept clearer.

According to Bernoulli’s law the pressure at the top of the cylinder is $-\frac{1}{2}\rho(V_0 + \delta U)^2$ and at the bottom it is $-\frac{1}{2}\rho(V_0 - \delta U)^2$, so the top to bottom difference in pressure $\Delta p = \frac{1}{2}\rho(4V_0\delta U)$. Therefore the magnitude of the upward lift force is

$$Lift\ Force = \frac{\Delta p}{2R} = \rho V_0 \frac{dU}{dz} \tag{1}$$

In terms of the given constants.

3. Discussion

The calculation of the lift force in (1) has been carried out by analogy with the recent calculation of the formula for the Magnus force on a spinning cylinder translating in a uniform fluid [1], which surprisingly has not been done before either.

What to put in for V_0 in order to evaluate the lift force in Equation (1) is the remaining question. In the case of a uniform flow past a cylinder a new theory has been put forward that does not depend on the irrotational assumption [1]. It predicts that at the top and bottom of the cylinder the relative fluid speed is 2.3 U, where U is the speed far from the cylinder. Using that value in (1), instead of the traditional 2U, would give a 15% higher lift force, where U is the speed far from the cylinder along a straight line that passes through the cylinder’s center.

From the sailor’s experience in the submarine surfacing in the wave zone came the idea some years ago for a wave pump [2] in which a straight piece of pipe, open at both ends, is held vertical below the sea surface and the waves passing by pump water up the pipe by means of the greater Bernoulli suction at the pipe’s top opening. This has been shown to work on a small scale in a swimming pool and some Japanese scientists are interested in applying the idea to farming the desert regions of the open ocean.

Reference

[1] Kenyon, K. E. (2016) On the Magnus Effect. Natural Science, **8**, 49-52.

[2] Kenyon, K. E. (2007) Upwelling by a wave pump. Journal of Oceanography, **63**, 327-331.